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#60 Statistical Methods (II)

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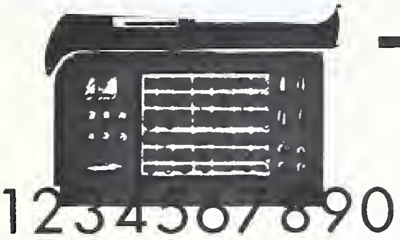

Determination of the number of REPLICATIONS REQUIRED IN RANDOMIZED BLOCK EXPERIMENTS

and the sensitivity of the F-test

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FOREWORD

Replication is considered essential in modern experimentation, and its importance to the accuracy of results is generally recognized. The accuracy of experiments can be enhanced, to some degree, by increasing the size of sample, or plot; but the most effective method is to increase the number of replications. The question, therefore, naturally arises as to the number of replications necessary to attain a specified level of accuracy.

Numerous inquiries, relating to the number of replications necessary for their proposed experiments, from research personnel of ARS have prompted the preparation of this article.

Copies of this article can be obtained from Biometrical Services, Agricultural Research Service, Plant Industry Station, Beltsville, Md.

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DETERMINATION OF THE NUMBER OF REPLICATIONS REQUIRED IN RANDOMIZED BLOCK EXPERIMENTS AND THE SENSITIVITY OF THE F-TEST

Judson U. McGuire[1]

The importance of increasing the number of replications is in decreasing the variance of a treatment mean, which is estimated by dividing the error mean square by the number of replications. It is obvious, therefore, that with sufficient numbers of replications any two treatment means, which are not identical, can be shown to be different in a statistical sense.

Because of the limitations of the available tables, the experimenter is in need of a procedure for determining the number of replications, with a minimum of effort.

The tables in this article[2] were constructed for the purpose of helping the experimenter decide, with a minimum amount of computation, the number of replications needed in a randomized block experiment to attain a specified degree of precision.

USE OF THE TABLES

Table 1 applies only to randomized block designs for two to nine means, at the 5-percent level of significance, which may be used in two ways: (1) To determine the number of replications needed to attain a specified probability of detecting a chosen difference, and (2) to determine the sensitivity of the F-test where, after analysis of a set of data, the treatments were found to be different. Both types of applications will be explained in detail.

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[2] These tables are based on Tang's tables of the power function of the analysis of variance test as given in the "Design and Analysis of Experiments" by O. Kempthorne. A graphic approach was employed, and for that reason they are not exact. A more accurate and extensive set of tables is being computed and will be published in a technical journal.

Determination of the Number of Replications

For the investigator to successfully determine the number of replications to be used in a randomized block experiment he must have a number of facts and estimates about the experiment and procedure. Methods for obtaining this information and its use in entering the tables will be discussed in the following order:

1. Magnitude of the difference between means to be tested,
2. Estimate of the experimental error, and
3. Stepwise example for using tables.

Magnitude of Mean Differences

The desired maximum difference between any two means may be based on prior experimental evidence, but may also be arbitrarily selected. The basis for the establishment of this difference may be (1) economic, (2) a change in quality or grade, or (3) an increase in the effectiveness of crop protection by some chemical compound or mixture of compounds.

Economic

Much applied research must be interpreted from an economic viewpoint. Thus, in certain types of field investigations it becomes necessary to establish a minimum quantity of bushels of a crop required to make a certain variety or treatment economically desirable. For example, a treatment may cost $5.00 an acre for materials and labor of application. If the crop sells for $1.75 a bushel and a return of 10% on the investment is desired, the minimum increase in yield required would be 3.14 bushels. (The cost of the treatment plus 10% return would be $5.50 which, when divided by the sale price of $1.75 per bushel, gives 3.14 bushels.)

Quality or Grade

Under some conditions a treatment may affect only the quality or grade of the experimental material. This is the case in the control of certain insects, such as the rice stinkbug, the aphids on cotton, or the scales on citrus, and plant pathogenic fungi or bacteria, such as the citrus canker of oranges or the scab of potatoes. Likewise, certain handling procedures at harvest affect the quality of such crops as potatoes, sweet potatoes, and sugar beets. Thus, if the crop is graded and yield calculated on that basis, as for example in tobacco, it becomes necessary for the investigator to determine the difference in percentage of top-most grade, or appropriate combination of grades, which may be applicable for the type of research under consideration.

Increase in Effectiveness of Treatment

Many of the new organic agricultural pesticides are specific for a pest or group of closely related pests. A direct consequence of this specificity is that a treatment may consist of several different materials applied together or serially. Usually the question asked is not with regard to the treatment which has been shown to be necessary, but with the effectiveness of the individual compound to kill a specific organism. Yield is generally of secondary importance, only the insecticidal or fungicidal property of some compound is to be measured. In this case the required difference must be stated in terms of percent kill. Following the above examples the research worker will be able to determine the required difference between two treatment means once the need is established.

Estimation of Experimental Error

Another prerequisite necessary for the determination of the required number of replications to be used in a particular experiment is an estimate of the error mean square from prior experimental work. If an estimate of error mean square is not available, based on the size of plot in the experiment being planned, then by use of Table 2 one can adjust an estimate from experiments comprising other plot sizes to that under consideration. For example: Two experiments are being planned—one, a preliminary test with 1/40th acre plots, and a second larger experiment with 1/5th acre plots. The only available estimate of the error mean square (V) is 13.6 bushels based on 9 degrees of freedom from an experiment composed of 1/10th acre plots. The ratios of the 1/40th and 1/5th acre plots to the 1/10th acre plots, respectively, are:

$$1/40 \div 1/10 = 1/4 = 1/n, \qquad \text{and}$$

$$1/5 \div 1/10 = 2 = n,$$

where n is the number of smaller plots comprising the larger plot, so that the 1/40th acre plot is 1/4th the area of the 1/10th acre plot and the 1/5th acre plot is twice the 1/10th acre plot.

The solution of

$$(1) \qquad V_l = (n/n^b)\,V \qquad \text{or} \qquad V_l = (2/2^b)\,V,$$

where V_s is the variance of the smaller plot and V_l is the variance of the larger plot, will give the value of the error mean square of the larger plot, (1/5th acre), while the solution of its inverse,

$$(2) \qquad V\left(\frac{1}{n/n^b}\right) = V_s \qquad \text{or} \qquad V\left(\frac{1}{4/4^b}\right) = V_s,$$

will give the value of the error mean square of the smaller plot

(1/40th acre). In the above situation, the equations (1) and (2) are solved by substituting the known variances and the values of n/n^b found opposite the appropriate n in Table 2. For the larger plot, the solution of equation (1) at $b = 0.5$ is

$$V_l = (1.41)(13.6)$$

$$= 19.18$$

while for equation (2), the solution is

$$V_s = (1/2)(13.6)$$

$$= 6.80.$$

A second adjustment which should be made on the error mean square (V_s) consists of a sampling adjustment based on the degrees of freedom corresponding to the error mean square. The error mean square used in the previous example was based on 9 degrees of freedom. Therefore, in order to allow for sampling errors the two adjusted error mean squares previously calculated should be multiplied by the factor for 9 degrees of freedom in Table 3. The adjusted two error mean squares are:

for the 1/5th acre plot	$41.43 = (2.16)(19.18)$
for the 1/40th acre plot	$14.43 = (2.16)(6.80)$

The above adjusted error mean squares should be used in computing the ratio of D/s, where D/s is an estimate of D/σ, needed for entry into Table 1.

Stepwise example for using tables

The use of the three tables will be best explained by outlining the following steps:

1. Establish the desired difference between any two means = D.

2. Obtain an estimate of the error mean square for the experimental material on a single plot basis = s^2.

3. Adjust the available mean square for:

   (a) Sampling error. Select the factor from Table 3 corresponding to the number of degrees of freedom upon which the estimated mean square is based and multiply the estimate of the mean square by it.

   (b) Difference in size of plot. Use Table 2 to get value of n/n^b and solve the appropriate equation on page (5).

4. Extract the square root of the adjusted variance.

5. Compute the ratio D/s, which is an estimate of D/σ.

6. Select that part of Table 1 for the appropriate number of means in the experiment.

7. Select the probability of detecting the chosen difference between any two means and proceed down that column until the value of D/s computed in (5) above is reached. Read off the number of replications required on the left margin of the table. If the exact value of D/s is not present, use the number of replications opposite the smaller value of D/σ in the table.

For example, five treatments are to be applied to corn for the control of the European corn borer. DDT is to be used as the standard, and four new insecticides are to be compared with it. The plots will be one tenth of an acre. An estimate of the variance per plot (s^2) of 0.086 based on 15 degrees of freedom is available. The difference between any two means (D) desired is 0.5 bushel per plot or 5 bushels to the acre. The information required for steps (1) and (2) is therefore available. In Table 3 we find, under 15 degrees of freedom, a value of 1.76 as the necessary factor for step (3, a). The adjusted mean square is determined as (1.76) (0.086) = 0.1522. The square root of 0.1522 = 0.39, which completes step 4. For step 5, D/s = 0.5/0.39 = 1.28. Therefore, referring to Table 1 (D), for five means, the following information is obtained:

Probability of detecting a given difference	Number of replications
0.50	8
0.60	9
0.70	11
0.80	13
0.90	16
0.95	19
0.99	26

In many instances the first choice of the probability for detecting a given difference between any two means is too stringent in that too many replications are required. In such cases, the experimenter must either (1) revise the value of D, (2) lower the probability of detecting a given difference, or (3) postpone the experiment until more work can be done on the technical aspects of the experiment which will reduce the variance. It is up to the experimenter to decide how high a level of the probability for detecting a given difference he can, or wishes to use. From the above tabulation it appears that with 13 replications there is only an 80 percent chance of detecting the difference chosen. Let us assume that only eight replications can be put in. By fixing the number of replications at eight, however, we must be satisfied with a larger difference. The value of D at a probability of P = 0.80 is (1.68) (0.39) = 0.655 bushels per plot, where 1.68 is the value in the 80% column of Table 1 (1) for 8 replications and 0.39 is the upper 90% point of the standard error per plot as previously computed.

Sensitivity of the F-test

Up to this point, the primary concern has been the determination of the number of replications needed to detect some desired difference. If the analysis of a set of data culminates in a significant F-test the experimenter rejects the null hypothesis, that all the treatments are estimates of the same population mean, with the understanding that he may be in error a certain percentage of the time in so doing. However, if the null hypothesis cannot be rejected, can the alternate hypothesis that at least one mean is from a different population, be accepted? The answer to this question lies in the size of the probability of detecting a given difference found in Table 1.

The ratio D'/s_x, where D' = the actual difference between the two extreme means and s_x = the standard error of a mean, is now of interest. The value of D'/s_x is entered in Table 1, under the appropriate section and the row for the actual number of replications used in the experiment. The probability of detecting the observed difference can be obtained by interpolation. For example, in an experiment to test the effect of 8 nutrients on the yield of pecans based on five replications, the F-test was not significant. The question becomes, can we accept the null hypothesis and say that the treatments had no effect on the yields, or was the nonsignificant result due to too few replications? In this example, the maximum mean difference (D') was 28 pounds of pecans; the variance per plot was 2,308.0; and the variance of a mean 461.6. The standard error of a treatment mean was approximately 21.48; therefore $D'/s_x = 28/21.48 = 1.30$. In Table 1 (G) it is apparent, reading across the row for five replications, that the probability of detecting the observed difference (D') is less than 0.5. Therefore, we should not accept the null hypothesis since the test was not sensitive enough to really detect a difference if one was actually present. It is apparent, therefore, that more replications are needed in order to detect a difference of 28 pounds, if the two treatments are actually different.

Table 1. Values for D/σ

Number of Replications	Probability of detecting a given difference						
	0.50	0.60	0.70	0.80	0.90	0.95	0.99
A. Two Means							
2	5.70	7.30	8.50	10.30	12.10	14.50	22.00
3	3.00	3.50	4.11	4.80	5.60	6.30	7.74
4	2.16	2.51	2.89	3.34	3.80	4.34	5.20
5	1.76	2.01	2.30	2.63	3.03	3.46	4.10
6	1.53	1.74	1.98	2.28	2.60	2.92	3.44
7	1.37	1.56	1.78	2.01	2.28	2.57	3.00
8	1.25	1.43	1.63	1.84	2.05	2.33	2.71
9	1.17	1.33	1.50	1.69	1.93	2.14	2.47
10	1.10	1.24	1.41	1.59	1.80	2.00	2.28
11	1.05	1.18	1.33	1.50	1.71	1.89	2.14
12	1.01	1.13	1.26	1.44	1.64	1.80	2.02
13	0.97	1.08	1.20	1.38	1.56	1.71	1.92
14	0.94	1.05	1.16	1.33	1.49	1.64	1.85
15	0.92	1.02	1.13	1.30	1.44	1.56	1.78
20	0.81	0.92	1.02	1.15	1.26	1.39	1.54
25		0.85	0.92	1.04	1.17	1.27	1.39
30		0.80	0.86	0.98	1.09	1.19	1.30
35			0.82	0.93	1.04	1.13	1.23
40				0.87	1.00	1.07	1.17
B. Three Means							
2	4.70	5.60	6.54	7.55	8.70	10.00	12.40
3	2.67	3.07	3.58	4.05	4.71	5.28	6.27
4	2.02	2.30	2.57	3.00	3.42	3.83	4.47
5	1.67	1.90	2.08	2.44	2.80	3.14	3.62
6	1.47	1.66	1.88	2.13	2.44	2.70	3.12
7	1.32	1.50	1.70	1.92	2.16	2.40	2.88
8	1.22	1.38	1.56	1.76	1.96	2.20	2.51
9	1.14	1.28	1.44	1.62	1.84	2.03	2.33
10	1.07	1.20	1.36	1.52	1.73	1.90	2.17
11	1.02	1.14	1.28	1.44	1.64	1.80	2.05
12	0.98	1.09	1.22	1.38	1.56	1.72	1.94
13	0.94	1.04	1.17	1.32	1.50	1.64	1.85
14	0.91	1.01	1.12	1.27	1.43	1.57	1.78
15	0.88	0.98	1.08	1.24	1.38	1.50	1.72
20		0.86	0.96	1.08	1.20	1.32	1.48
25			0.86	0.98	1.10	1.19	1.34
30			0.80	0.91	1.01	1.10	1.24
35				0.85	0.96	1.04	1.17
40				0.80	0.91	0.98	1.10

Table 1. Values for D/σ (cont.)

Number of Replications	Probability of detecting a given difference						
	0.50	0.60	0.70	0.80	0.90	0.95	0.99
				C. Four Means			
2	4.08	4.71	5.50	6.20	7.01	7.95	9.26
3	2.48	2.81	3.25	3.63	4.22	4.70	5.42
4	1.92	2.19	2.50	2.80	3.20	3.54	4.02
5	1.62	1.84	2.08	2.32	2.66	2.95	3.36
6	1.44	1.62	1.81	2.05	2.33	2.57	2.95
7	1.30	1.47	1.65	1.85	2.08	2.30	2.62
8	1.19	1.35	1.51	1.71	1.90	2.12	2.41
9	1.12	1.26	1.40	1.58	1.79	1.97	2.25
10	1.05	1.17	1.33	1.48	1.68	1.89	2.10
11	0.99	1.12	1.26	1.40	1.60	1.75	1.98
12	0.96	1.06	1.19	1.35	1.53	1.67	1.88
13	0.92	1.02	1.14	1.28	1.46	1.59	1.81
14	0.89	0.98	1.09	1.24	1.40	1.53	1.74
15	0.86	0.95	1.05	1.20	1.34	1.46	1.68
20		0.82	0.92	1.03	1.16	1.27	1.45
25			0.822	0.93	1.05	1.14	1.31
30				0.86	0.96	1.05	1.20
35				0.805	0.901	0.98	1.13
40					0.86	0.92	1.06
				D. Five Means			
2	3.73	4.20	4.90	5.45	6.14	6.82	7.75
3	2.37	2.66	3.04	3.39	3.90	4.32	4.91
4	1.87	2.11	2.40	2.67	3.04	3.35	3.75
5	1.59	1.79	2.01	2.26	2.56	2.83	3.19
6	1.42	1.59	1.78	2.00	2.28	2.48	2.82
7	1.33	1.45	1.62	1.82	2.03	2.23	2.53
8	1.13	1.33	1.49	1.68	1.87	2.06	2.34
9	1.11	1.24	1.38	1.55	1.75	1.93	2.20
10	1.04	1.16	1.31	1.45	1.65	1.80	2.05
11	0.98	1.10	1.24	1.38	1.57	1.71	1.95
12	0.94	1.05	1.18	1.32	1.50	1.64	1.85
13	0.90	1.00	1.13	1.27	1.44	1.56	1.78
14	0.87	0.97	1.08	1.21	1.37	1.50	1.72
15	0.85	0.935	1.03	1.17	1.32	1.45	1.66
20		0.795	0.891	1.01	1.14	1.24	1.43
25			0.797	0.90	1.02	1.11	1.29
30				0.837	0.928	1.02	1.18
35					0.865	0.95	1.10
40					0.82	0.88	1.04

Table 1. Values for D/σ (cont.)

Number of Replications	Probability of detecting a given difference						
	0.50	0.60	0.70	0.80	0.90	0.95	0.99
E. Six Means							
2	3.50	3.90	4.50	5.00	5.60	6.18	6.90
3	2.31	2.55	2.91	3.22	3.70	4.06	4.57
4	1.84	2.05	2.33	2.58	2.94	3.21	3.58
5	1.56	1.76	1.99	2.21	2.50	2.74	3.07
6	1.40	1.57	1.75	1.96	2.22	2.42	2.74
7	1.26	1.44	1.60	1.79	2.00	2.19	2.48
8	1.17	1.31	1.48	1.66	1.85	2.03	2.30
9	1.10	1.23	1.37	1.54	1.74	1.90	2.15
10	1.03	1.15	1.29	1.44	1.63	1.78	2.02
11	0.98	1.09	1.23	1.37	1.55	1.69	1.92
12	0.93	1.04	1.17	1.31	1.48	1.61	1.84
13	0.89	0.99	1.12	1.25	1.42	1.55	1.74
14	0.86	0.96	1.06	1.20	1.35	1.49	1.70
15	0.84	0.92	1.02	1.16	1.30	1.43	1.65
20			0.872	0.99	1.12	1.22	1.42
25				0.88	1.00	1.08	1.27
30				0.80	0.902	0.99	1.16
35					0.838	0.92	1.08
40						0.85	1.02
F. Seven Means							
2	3.34	3.70	4.23	4.72	5.30	5.77	6.44
3	2.26	2.50	2.82	3.12	3.58	3.88	4.33
4	1.82	2.02	2.29	2.51	2.87	3.11	3.45
5	1.55	1.75	1.96	2.18	2.45	2.68	2.98
6	1.40	1.56	1.74	1.94	2.20	2.38	2.69
7	1.26	1.43	1.59	1.77	1.98	2.16	2.43
8	1.16	1.31	1.46	1.65	1.84	2.00	2.27
9	1.09	1.22	1.36	1.52	1.72	1.88	2.12
10	1.02	1.14	1.29	1.43	1.62	1.76	2.00
11	0.96	1.08	1.22	1.35	1.54	1.67	1.91
12	0.92	1.03	1.16	1.30	1.47	1.60	1.82
13	0.89	0.99	1.11	1.24	1.41	1.54	1.76
14	0.86	0.95	1.06	1.19	1.35	1.47	1.69
15	0.83	0.91	1.01	1.15	1.29	1.42	1.64
20			0.86	0.97	1.10	1.20	1.41
25				0.86	0.98	1.07	1.27
30					0.885	0.97	1.15
35					0.82	0.89	1.07
40						0.83	1.00

Table 1. Values for D/σ (cont.)

Number of Replications	Probability of detecting a given difference						
	0.50	0.60	0.70	0.80	0.90	0.95	0.99
G. Eight Means							
2	3.20	3.58	4.06	4.56	5.12	5.51	6.16
3	2.24	2.47	2.78	3.07	3.49	3.75	4.17
4	1.80	2.00	2.26	2.50	2.80	3.05	3.38
5	1.55	1.74	1.95	2.17	2.41	2.63	2.92
6	1.40	1.56	1.72	1.93	2.18	2.35	2.64
7	1.27	1.42	1.59	1.76	1.97	2.14	2.41
8	1.16	1.30	1.46	1.64	1.83	2.00	2.24
9	1.09	1.22	1.36	1.52	1.71	1.87	2.11
10	1.03	1.14	1.28	1.42	1.61	1.75	2.00
11	0.96	1.08	1.22	1.35	1.52	1.66	1.90
12	0.92	1.03	1.16	1.29	1.46	1.59	1.82
13	0.89	0.98	1.10	1.24	1.40	1.52	1.75
14	0.86	0.94	1.05	1.18	1.34	1.47	1.69
15	0.82	0.905	1.00	1.14	1.29	1.41	1.63
20			0.85	0.96	1.09	1.19	1.40
25				0.846	0.97	1.05	1.26
30					0.87	0.95	1.14
35					0.80	0.88	1.06
40						0.81	0.99
H. Nine Means							
2	3.14	3.51	3.94	4.42	5.00	5.37	6.00
3	2.20	2.45	2.74	3.03	3.44	3.66	4.05
4	1.78	2.00	2.24	2.48	2.76	3.00	3.32
5	1.54	1.74	1.94	2.16	2.40	2.60	2.90
6	1.38	1.55	1.71	1.91	2.16	2.32	2.62
7	1.26	1.42	1.58	1.75	1.96	2.12	2.40
8	1.15	1.30	1.46	1.64	1.82	1.98	2.12
9	1.08	1.22	1.36	1.52	1.70	1.86	2.10
10	1.02	1.14	1.28	1.42	1.60	1.75	2.00
11	0.95	1.07	1.22	1.35	1.52	1.66	1.90
12	0.90	1.03	1.16	1.29	1.46	1.58	1.81
13	0.86	0.98	1.10	1.24	1.40	1.52	1.75
14	0.82	0.94	1.05	1.18	1.34	1.46	1.68
15	0.80	0.90	1.00	1.14	1.28	1.40	1.62
20			0.84	0.95	1.08	1.18	1.40
25				0.837	0.96	1.04	1.25
30					0.86	0.94	1.14
35						0.87	1.05
40						0.80	0.98

Table 2. The values of n/n^b for $b = 0.05$

N	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	1.00	1.04	1.09	1.14	1.18	1.22	1.25	1.30	1.34	1.37
2	1.41	1.44	1.48	1.51	1.54	1.58	1.61	1.64	1.67	1.70
3	1.73	1.76	1.78	1.81	1.84	1.87	1.89	1.92	1.94	1.97
4	2.00	2.02	2.04	2.07	2.09	2.11	2.14	2.16	2.19	2.21
5	2.23					2.34				
6	2.44					2.54				
7	2.64					2.73				
8	2.82					2.91				
9	3.00					3.08				
10	3.16									

Table 3. Values of the 90 percent upper point in the distribution of the sample variance. $P(\sigma^2 < s^2 m/x^2) = 0.90$, where m is the number of degrees of freedom.

(m = d.f.)	0	1	2	3	4	5	6	7	8	9
0		63.29	9.48	5.14	3.76	3.11	2.72	2.47	2.29	2.16
10	2.06	1.97	1.90	1.85	1.80	1.76	1.72	1.69	1.66	1.63
20	1.61	1.59	1.57	1.55	1.53	1.52	1.50	1.49	1.48	1.47
30	1.46					1.42				
40	1.38					1.36				
50	1.33					1.31				
60	1.29									
120	1.19									
240	1.13									
1000	1.06									
∞	1.00									

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